

of the observations now in progress at the Arkley Observatory should form a sequel to the present communication, and thus afford a complete solution of the great problem of determining the solar parallax from the parallactic inequality in the motion of the Moon.

Arkley Observatory : Barnet, Herts,
1880, May 13.

Investigation of the Secular Acceleration of the Moon's Mean Motion, caused by the Secular Change in the Eccentricity of the Earth's Orbit; taking into Account Terms of the Order of m^4 , but neglecting the Eccentricity and Inclination of the Moon's Orbit.
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As the question of the Moon's secular acceleration has lately been again brought before the Society, I have thought that it might not be useless or without interest to communicate an investigation of the two leading terms of that acceleration which I gave many years ago in my lectures on the lunar theory.

1. Let r, θ be the polar coordinates of the Moon at time t , $u = \frac{1}{r}$, $H = r^2 \frac{d\theta}{dt}$, μ the sum of the masses of the Earth and Moon;

also let m' be the mass of the Sun, r', θ' its polar coordinates, a' the Sun's mean distance, n' its mean motion, and e' the eccentricity of its orbit, $\lambda' = n't + \epsilon'$ its mean longitude, and $\phi' = n't + \epsilon' - \omega'$ its mean anomaly.

Then the equations to be satisfied are

$$\frac{d^2 u}{d\theta^2} + u = \frac{\mu}{H^2} - \frac{1}{2} \frac{m'}{H^2 u^3 r'^3} - \frac{3}{2} \frac{m'}{H^2 u^3 r'^3} \cos 2(\theta - \theta') \\ + \frac{3}{2} \frac{m'}{H^2 u^4 r'^3} \frac{du}{d\theta} \sin 2(\theta - \theta'),$$

and

$$\frac{d(H^2)}{d\theta^2} = - \frac{3m'}{u^4 r'^3} \sin 2(\theta - \theta').$$

Also, by the formulæ of elliptic motion

$$\frac{m'}{r'^3} = \frac{m'}{a'^3} \left(\frac{a'}{r'} \right)^3 = n'^2 \left\{ 1 + \frac{3}{2} e'^2 + 3e' \cos \phi' + \frac{9}{2} e'^2 \cos 2\phi' \right\}, \\ \frac{m'}{r'^3} \cos 2(\theta - \theta') = \frac{m'}{a'^3} \left(\frac{a'}{r'} \right)^3 \cos 2(\theta - \theta') \\ = n'^2 \left\{ \left(1 - \frac{5}{2} e'^2 \right) \cos 2(\theta - \lambda') + \frac{7}{2} e' \cos (2\theta - 2\lambda' - \phi') \right. \\ \left. - \frac{1}{2} e' \cos (2\theta - 2\lambda' + \phi') \right. \\ \left. + \frac{17}{2} e'^2 \cos (2\theta - 2\lambda' - 2\phi') \right\},$$

and

$$\begin{aligned} \frac{m}{r'^3} \sin 2(\theta - \theta') &= \frac{m'}{a'^3} \left(\frac{a'}{r'} \right)^3 \sin 2(\theta - \theta') \\ &= n'^2 \left\{ \left(1 - \frac{5}{2} e'^2 \right) \sin 2(\theta - \lambda') + \frac{7}{2} e' \sin (2\theta - 2\lambda' - \phi') \right. \\ &\quad \left. - \frac{1}{2} e' \sin (2\theta - 2\lambda' + \phi') \right. \\ &\quad \left. + \frac{17}{2} e'^2 \sin (2\theta - 2\lambda' - 2\phi') \right\}. \end{aligned}$$

The angles involved in these expressions are formed by combining the angle $2\theta - 2\lambda'$ with multiples of ϕ' .

For our present purpose we may omit the terms which involve $2\phi'$. Also, for the sake of brevity we may write

$$\begin{aligned} n't &\text{ instead of } n't + \epsilon' - \omega' \text{ or } \phi', \\ 2\theta - 2n't &\text{ instead of } 2\theta - 2(n't + \epsilon') \text{ or } 2\theta - 2\lambda', \\ 2\theta - 3n't &\text{ instead of } 2\theta - 2\lambda' - \phi', \\ 2\theta - n't &\text{ instead of } 2\theta - 2\lambda + \phi', \end{aligned}$$

since no ambiguity can arise from this abbreviation.

Hence our equations become

$$\begin{aligned} \frac{d^2 u}{d\theta^2} + u &= \frac{\mu}{H^2} - \frac{1}{2} \frac{n'^2}{H^2 u^3} \left\{ 1 + \frac{3}{2} e'^2 + 3e' \cos n't \right\} \\ &\quad - \frac{3}{2} \frac{n'^2}{H^2 u^3} \left\{ \left(1 - \frac{5}{2} e'^2 \right) \cos (2\theta - 2n't) + \frac{7}{2} e' \cos (2\theta - 3n't) \right. \\ &\quad \left. - \frac{1}{2} e' \cos (2\theta - n't) \right\} \\ &\quad + \frac{3}{2} \frac{n'^2}{H^2 u^4} \frac{du}{d\theta} \left\{ \left(1 - \frac{5}{2} e'^2 \right) \sin (2\theta - 2n't) + \frac{7}{2} e' \sin (2\theta - 3n't) \right. \\ &\quad \left. - \frac{1}{2} e' \sin (2\theta - n't) \right\}, \end{aligned}$$

and

$$\begin{aligned} \frac{d(H^2)}{d\theta} &= -\frac{3n'^2}{u^4} \left\{ \left(1 - \frac{5}{2} e'^2 \right) \sin (2\theta - 2n't) + \frac{7}{2} e' \sin (2\theta - 3n't) \right. \\ &\quad \left. - \frac{1}{2} e' \sin (2\theta - n't) \right\}. \end{aligned}$$

2. After these preliminaries, it will be convenient to begin by finding the relations between the actual mean motion n of the Moon and the constant parts of u and H^2 when these quantities are developed in the form we have adopted, carrying the approximation as far as terms involving $m^4 e'^2$, on the supposition that e' and therefore also that n is constant.

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For this purpose it is sufficient to take

$$\begin{aligned} nt + \epsilon &= \theta + 3me' \sin n't - \frac{11}{8}m^2 \left(1 - \frac{5}{2}e'^2\right) \sin (2\theta - 2n't) \\ &\quad - \frac{77}{16}m^2e' \sin (2\theta - 3n't) + \frac{11}{16}m^2e' \sin (2\theta - n't), \\ u &= \frac{1}{a} \left\{ 1 - \frac{3}{2}m^2e' \cos n't + m^2 \left(1 - \frac{5}{2}e'^2\right) \cos (2\theta - 2n't) \right. \\ &\quad \left. + \frac{7}{2}m^2e' \cos (2\theta - 3n't) - \frac{1}{2}m^2e' \cos (2\theta - n't) \right\}, \end{aligned}$$

which are readily derived from the equations of motion.

Differentiate the first of these equations and put

$$\frac{n'}{n} = m,$$

$$\begin{aligned} \therefore \frac{ndt}{d\theta} &\left\{ 1 - 3m^2e' \cos n't - \frac{11}{4}m^3 \left(1 - \frac{5}{2}e'^2\right) \cos (2\theta - 2n't) - \frac{231}{16}m^3e' \cos (2\theta - 3n't) \right. \\ &\quad \left. + \frac{11}{16}m^3e' \cos (2\theta - n't) \right\} \\ &= 1 - \frac{11}{4}m^2 \left(1 - \frac{5}{2}e'^2\right) \cos (2\theta - 2n't) - \frac{77}{8}m^2e' \cos (2\theta - 3n't) + \frac{11}{8}m^2e' \cos (2\theta - n't), \end{aligned}$$

or

$$\begin{aligned} \frac{ndt}{d\theta} &= 1 + \frac{9}{2}m^4e'^2 + 3m^2e' \cos n't - \frac{11}{4}m^2 \left(1 - \frac{5}{2}e'^2\right) \cos (2\theta - 2n't) \\ &\quad - \frac{77}{8}m^2e' \cos (2\theta - 3n't) + \frac{11}{8}m^2e' \cos (2\theta - n't), \end{aligned}$$

since the other terms only give rise to terms of higher orders than we have here taken into account.

Hence

$$\begin{aligned} H^2 &= \left(\frac{d\theta}{u^2 dt} \right)^2 = u^{-4} \left(\frac{dt}{d\theta} \right)^{-2} \\ &= n^2 a^4 \left\{ 1 + 5m^4(1 - 5e'^2) + \frac{45}{4}m^4e'^2 + \frac{245}{4}m^4e'^2 + \frac{5}{4}m^4e'^2 + 6m^2e' \cos n't \right. \\ &\quad \left. - 4m^2 \left(1 - \frac{5}{2}e'^2\right) \cos (2\theta - 2n't) - 14m^2e' \cos (2\theta - 3n't) + 2m^2e' \cos (2\theta - n't) \right\} \\ &\times \left\{ 1 - 9m^4e'^2 + \frac{27}{2}m^4e'^2 + \frac{363}{32}m^4(1 - 5e'^2) + \frac{17787}{128}m^4e'^2 + \frac{363}{128}m^4e'^2 \right. \\ &\quad \left. - 6m^2e' \cos n't + \frac{11}{2}m^2 \left(1 - \frac{5}{2}e'^2\right) \cos (2\theta - 2n't) + \frac{77}{4}m^2e' \cos (2\theta - 3n't) \right. \\ &\quad \left. - \frac{11}{4}m^2e' \cos (2\theta - n't) \right\}; \end{aligned}$$

or, by actual multiplication,

$$\begin{aligned} H^2 &= n^2 a^4 \left\{ 1 + \frac{523}{32} m^4 (1 - 5e'^2) + \frac{14083}{64} m^4 e'^2 - 18 m^4 e'^2 - 11 m^4 (1 - 5e'^2) \right. \\ &\quad \left. - \frac{539}{4} m^4 e'^2 - \frac{11}{4} m^4 e'^2 + \frac{3}{2} m^2 \left(1 - \frac{5}{2} e'^2 \right) \cos (2\theta - 2n't) \right. \\ &\quad \left. + \frac{21}{4} m^2 e' \cos (2\theta - 3n't) - \frac{3}{4} m^2 e' \cos (2\theta - n't) \right\} \\ &= n^2 a^4 \left\{ 1 + \frac{171}{32} m^4 (1 - 5e'^2) + \frac{4131}{64} m^4 e'^2 + \frac{3}{2} m^2 \left(1 - \frac{5}{2} e'^2 \right) \cos (2\theta - 2n't) \right. \\ &\quad \left. + \frac{21}{4} m^2 e' \cos (2\theta - 3n't) - \frac{3}{4} m^2 e' \cos (2\theta - n't) \right\}. \end{aligned}$$

Hence the constant part of H^2 is

$$n^2 a^4 \left\{ 1 + \frac{171}{32} m^4 + \frac{2421}{64} m^4 e'^2 \right\},$$

n being the actual mean motion.

Hence

$$\begin{aligned} \frac{\mu}{H^2} &= \frac{\mu}{n^2 a^4} \left\{ 1 - \frac{171}{32} m^4 (1 - 5e'^2) - \frac{4131}{64} m^4 e'^2 + \frac{9}{8} m^4 (1 - 5e'^2) + \frac{441}{32} m^4 e'^2 + \frac{9}{32} m^4 e'^2 \right. \\ &\quad \left. - \frac{3}{2} m^2 \left(1 - \frac{5}{2} e'^2 \right) \cos (2\theta - 2n't) - \frac{21}{4} m^2 e' \cos (2\theta - 3n't) \right. \\ &\quad \left. + \frac{3}{4} m^2 e' \cos (2\theta - n't) \right\} \\ &= \frac{\mu}{n^2 a^4} \left\{ 1 - \frac{135}{32} m^4 (1 - 5e'^2) - \frac{3231}{64} m^4 e'^2 - \frac{3}{2} m^2 \left(1 - \frac{5}{2} e'^2 \right) \cos (2\theta - 2n't) \right. \\ &\quad \left. - \frac{21}{4} m^2 e' \cos (2\theta - 3n't) + \frac{3}{4} m^2 e' \cos (2\theta - n't) \right\}, \end{aligned}$$

and therefore the constant part of $\frac{\mu}{H^2}$ is

$$\frac{\mu}{n^2 a^4} \left\{ 1 - \frac{135}{32} m^4 - \frac{1881}{64} m^4 e'^2 \right\}.$$

3. Also

$$\begin{aligned} \frac{du}{d\theta} &= \frac{1}{a} \left\{ \frac{3}{2} m^3 e' \sin n't - 2 m^2 \left(1 - \frac{5}{2} e'^2 \right) \sin (2\theta - 2n't) \right. \\ &\quad \left. - 7 m^2 e' \sin (2\theta - 3n't) + m^2 e' \sin (2\theta - n't) \right\}, \end{aligned}$$

and

$$\begin{aligned} \frac{d^2 u}{d\theta^2} &= \frac{1}{a} \left\{ -4 m^2 \left(1 - \frac{5}{2} e'^2 \right) \cos (2\theta - 2n't) - 14 m^2 e' \cos (2\theta - 3n't) \right. \\ &\quad \left. + 2 m^2 e' \cos (2\theta - n't) \right\}; \\ &\quad \text{L L 2} \end{aligned}$$

constant (or rather the non-periodic) parts of u and of H^2 may have the same forms as before, and in this case we shall find the same relation between n and a as that which has just been found, and n will continue to signify the *actual mean motion* at the time to which θ belongs, but n and a will now become variable quantities, and, in order to satisfy our equations, it will be necessary to add certain periodic terms to u and H^2 which would not exist if e' were constant.

Suppose then that

$$u = \frac{1}{a} \left\{ 1 + \delta v - \frac{3}{2} m^2 e' \cos n't + m^2 \left(1 - \frac{5}{2} e'^2 \right) \cos (2\theta - 2n't) + \frac{7}{2} m^2 e' \cos (2\theta - 3n't) - \frac{1}{2} m^2 e' \cos (2\theta - n't) \right\},$$

and

$$H^2 = n^2 a^4 \left\{ 1 + 2\delta\eta + \frac{171}{32} m^4 + \frac{2421}{64} m^4 e'^2 + \frac{3}{2} m^2 \left(1 - \frac{5}{2} e'^2 \right) \cos (2\theta - 2n't) + \frac{21}{4} m^2 e' \cos (2\theta - 3n't) - \frac{3}{4} m^2 e' \cos (2\theta - n't) \right\}.$$

We will suppose e' to vary uniformly with the time, and very slowly, or, in other words, we will suppose

$$\frac{de'}{dt} \text{ to be constant, so that } \frac{d^2 e'}{dt^2} = 0,$$

and we will neglect

$$\left(\frac{de'}{dt} \right)^2.$$

We must therefore recollect that $\frac{de'}{d\theta}$ is not constant, but is equal to

$$\begin{aligned} \frac{de'}{dt} \cdot \frac{dt}{d\theta} &= \frac{1}{Hu^2} \cdot \frac{de'}{dt} \\ &= \frac{de'}{ndt} \left\{ 1 + 3m^2 e' \cos n't - \frac{11}{4} m^2 \cos (2\theta - 2n't) - \frac{77}{8} m^2 e' \cos (2\theta - 3n't) + \frac{11}{8} m^2 e' \cos (2\theta - n't) \right\}. \end{aligned}$$

5. In consequence of the variability of e' , $\frac{du}{dt}$ will contain the additional terms

$$\begin{aligned} \frac{1}{Hu^2} \cdot \frac{1}{a} \left\{ -\frac{da}{adt} - \frac{3}{2} m^2 \frac{de'}{dt} \cos n't - 5m^2 e' \frac{de'}{dt} \cos (2\theta - 2n't) + \frac{7}{2} m^2 \frac{de'}{dt} \cos (2\theta - 3n't) - \frac{1}{2} m^2 \frac{de'}{dt} \cos (2\theta - n't) \right\} \\ + \frac{1}{a} \cdot \frac{d \cdot \delta v}{d\theta}, \end{aligned}$$

or

$$\frac{1}{an} \left\{ -\frac{da}{adt} - \frac{3}{2} m^2 \frac{de'}{dt} \cos n't - 5m^2 e' \frac{de'}{dt} \cos (2\theta - 2n't) + \frac{7}{2} m^2 \frac{de'}{dt} \cos (2\theta - 3n't) \right. \\ \left. - \frac{1}{2} m^2 \frac{de'}{dt} \cos (2\theta - n't) \right\} \\ + \frac{1}{a} \cdot \frac{d \cdot \delta v}{d\theta},$$

to the order of approximation required.

Therefore also $\frac{d^2 u}{d\theta^2}$ will contain the additional terms

$$\frac{1}{an} \left\{ 10m^2 e' \frac{de'}{dt} \sin (2\theta - 2n't) - 7m^2 \frac{de'}{dt} \sin (2\theta - 3n't) + m^2 \frac{de'}{dt} \sin (2\theta - n't) \right. \\ \left. + 10m^2 e' \frac{de'}{dt} \sin (2\theta - 2n't) - 7m^2 \frac{de'}{dt} \sin (2\theta - 3n't) + m^2 \frac{de'}{dt} \sin (2\theta - n't) \right\} \\ + \frac{1}{a} \frac{d^2 \cdot \delta v}{d\theta^2}$$

neglecting

$$\frac{d^2 a}{dt^2}, \left(\frac{da}{dt} \right)^2, \text{ and also } m^3 \frac{de'}{dt}$$

in the coefficients of the periodic terms.

Hence

$$\frac{d^2 u}{d\theta^2} + u$$

contains the additional terms

$$\frac{1}{a} \left\{ \frac{d^2 \cdot \delta v}{d\theta^2} + \delta v \right\} \\ + \frac{1}{an} \left\{ 20m^2 e' \frac{de'}{dt} \sin (2\theta - 2n't) - 14m^2 \frac{de'}{dt} \sin (2\theta - 3n't) + 2m^2 \frac{de'}{dt} \sin (2\theta - n't) \right\}.$$

Also $\frac{\mu}{H^2}$ contains the additional term

$$\frac{\mu}{n^2 a^4} [-2\delta\eta].$$

The other terms which enter into the first differential equation receive no additional terms of the order to which we restrict ourselves.

6. Also differentiating the expression for H^2 , and including terms of the order

$$m^4 e' \frac{de'}{dt}$$

in the non-periodic part, but only those of the orders

$$m^2 \frac{de'}{dt} \text{ and } m^2 e' \frac{de'}{dt}$$

in the periodic part, we have the following additional terms in $\frac{d(H^2)}{d\theta}$, viz.

$$\begin{aligned} n^2 a^4 \frac{1}{H a^2} \left\{ \frac{2dn}{ndt} + \frac{4da}{adt} + \frac{2421}{32} m^4 e' \frac{de'}{dt} - \frac{15}{2} m^2 e' \frac{de'}{dt} \cos(2\theta - 2n't) \right. \\ \left. + \frac{21}{4} m^2 \frac{de'}{dt} \cos(2\theta - 3n't) - \frac{3}{4} m^2 \frac{de'}{dt} \cos(2\theta - n't) \right\} \\ + n^2 a^4 \left\{ 2 \frac{d \cdot \delta\eta}{d\theta} \right\}. \end{aligned}$$

Also the right-hand side of the second differential equation contains the following additional quantity:—

$$m^2 n^2 a^4 [4\delta v] \left\{ 3 \sin(2\theta - 2n't) + \frac{21}{2} e' \sin(2\theta - 3n't) - \frac{3}{2} e' \sin(2\theta - n't) \right\},$$

which, as we shall immediately find, contains non-periodic terms of the order

$$m^4 e' \frac{de'}{dt}.$$

Hence, taking the periodic parts of this equation, we have

$$2 \frac{d \cdot \delta\eta}{d\theta} = \frac{1}{n} \left\{ \frac{15}{2} m^2 e' \frac{de'}{dt} \cos(2\theta - 2n't) - \frac{21}{4} m^2 \frac{de'}{dt} \cos(2\theta - 3n't) \right. \\ \left. + \frac{3}{4} m^2 \frac{de'}{dt} \cos(2\theta - n't) \right\};$$

$$\therefore 2(\delta\eta) = \frac{1}{n} \left\{ \frac{15}{4} m^2 e' \frac{de'}{dt} \sin(2\theta - 2n't) - \frac{21}{8} m^2 \frac{de'}{dt} \sin(2\theta - 3n't) \right. \\ \left. + \frac{3}{8} m^2 \frac{de'}{dt} \sin(2\theta - n't) \right\}.$$

7. Substitute this in the first equation, putting

$$\frac{\mu}{n^2 a^3} = 1$$

in the coefficients of the periodic terms, as these are only required to the order of m^2 , and we obtain

$$\begin{aligned} \frac{d^2 \cdot \delta v}{d\theta^2} + \delta v &= -\frac{1}{n} \left\{ 20 m^2 e' \frac{de'}{dt} \sin(2\theta - 2n't) - 14 m^2 \frac{de'}{dt} \sin(2\theta - 3n't) \right. \\ &\quad + 2 m^2 \frac{de'}{dt} \sin(2\theta - n't) + \frac{15}{4} m^2 e' \frac{de'}{dt} \sin(2\theta - 2n't) \\ &\quad \left. - \frac{21}{8} m^2 \frac{de'}{dt} \sin(2\theta - 3n't) + \frac{3}{8} m^2 \frac{de'}{dt} \sin(2\theta - n't) \right\} \\ &= -\frac{1}{n} \left\{ \frac{95}{4} m^2 e' \frac{de'}{dt} \sin(2\theta - 2n't) - \frac{133}{8} m^2 \frac{de'}{dt} \sin(2\theta - 3n't) \right. \\ &\quad \left. + \frac{19}{8} m^2 \frac{de'}{dt} \sin(2\theta - n't) \right\}. \end{aligned}$$

$$\therefore \delta v = \frac{1}{n} \left\{ \frac{95}{12} m^2 e' \frac{de'}{dt} \sin(2\theta - 2n't) - \frac{133}{24} m^2 \frac{de'}{dt} \sin(2\theta - 3n't) \right. \\ \left. + \frac{19}{24} m^2 \frac{de'}{dt} \sin(2\theta - n't) \right\}.$$

Substitute this value of δv , and also the value of $\frac{1}{Hu^2}$, viz.

$$\frac{1}{n} \left\{ 1 + 3m^2e' \cos n't - \frac{11}{4} m^2 \cos (2\theta - 2n't) - \frac{77}{8} m^2e' \cos (2\theta - 3n't) + \frac{11}{8} m^2e' \cos (2\theta - n't) \right\},$$

for that quantity in the second differential equation, and equate the non-periodic parts which result from this substitution,

$$\begin{aligned} \therefore \frac{2dn}{ndt} + \frac{4da}{adt} + \frac{2421}{32} m^4e' \frac{de'}{dt} + \frac{165}{16} m^4e' \frac{de'}{dt} - \frac{1617}{64} m^4e' \frac{de'}{dt} - \frac{33}{64} m^4e' \frac{de'}{dt} \\ = \frac{95}{2} m^4e' \frac{de'}{dt} - \frac{931}{8} m^4e' \frac{de'}{dt} - \frac{19}{8} m^4e' \frac{de'}{dt}, \end{aligned}$$

or

$$\begin{aligned} \frac{2dn}{ndt} + 4 \frac{da}{adt} + \frac{963}{16} m^4e' \frac{de'}{dt} = -\frac{285}{4} m^4e' \frac{de'}{dt}, \\ \therefore \frac{dn}{ndt} + 2 \frac{da}{adt} = -\frac{2103}{32} m^4e' \frac{de'}{dt}. \end{aligned}$$

8. The substitution of the values of δv and $\delta \eta$ in the first differential equation introduces no non-periodic terms depending on $\frac{de'}{dt}$; consequently the value of $\frac{\mu}{n^2a^3}$ remains of the same form as before.

Hence

$$\begin{aligned} \log \left(\frac{\mu}{n^2a^3} \right) &= \frac{1}{2} m^2 \left(1 + \frac{3}{2} e'^2 \right) + \frac{75}{32} m^4 + \frac{1197}{64} m^4e'^2 - \frac{1}{8} m^4(1 + 3e'^2) \\ &= \frac{1}{2} m^2 \left(1 + \frac{3}{2} e'^2 \right) + \frac{71}{32} m^4 + \frac{1173}{64} m^4e'^2; \\ \therefore \frac{2dn}{ndt} + 3 \frac{da}{adt} &= -\frac{3}{2} m^2e' \frac{de'}{dt} - \frac{1173}{32} m^4e' \frac{de'}{dt} - m^2 \left(\frac{dm}{mdt} \right) \\ &= -\left(\frac{3}{2} m^2 + \frac{1173}{32} m^4 \right) e' \frac{de'}{dt} + m^2 \left(\frac{dn}{ndt} \right), \end{aligned}$$

since

$$m = \frac{n'}{n}, \text{ and } \therefore \frac{dm}{mdt} = -\frac{dn}{ndt},$$

n' being constant.

Hence

$$(4 - 2m^2) \frac{dn}{ndt} + 6 \frac{da}{adt} = -\left(3m^2 + \frac{1173}{16} m^4 \right) e' \frac{de'}{dt},$$

also from above

$$\begin{aligned} 3 \frac{dn}{ndt} + 6 \frac{da}{adt} &= -\frac{6309}{32} m^4e' \frac{de'}{dt}; \\ \therefore (1 - 2m^2) \frac{dn}{ndt} &= -\left(3m^2 - \frac{3963}{32} m^4 \right) e' \frac{de'}{dt}, \end{aligned}$$

and

$$\begin{aligned}\frac{dn}{ndt} &= -\left(3m^2 - \frac{3771}{32}m^4\right)e'\frac{de'}{dt}; \\ \therefore 2\frac{da}{adt} &= \left(3m^2 - \frac{3771}{32}m^4\right)e'\frac{de'}{dt} - \frac{2103}{32}m^4e'\frac{de'}{dt} \\ &= \left(3m^2 - \frac{2937}{16}m^4\right)e'\frac{de'}{dt},\end{aligned}$$

or

$$\frac{da}{adt} = \left(\frac{3}{2}m^2 - \frac{2937}{32}m^4\right)e'\frac{de'}{dt}.$$

9. These equations give the rate of variation of the quantities n and a . We will now show that n denotes the actual mean motion, as it did when e' was constant.

From the values of u and H^2 we find

$$\begin{aligned}\frac{dt}{d\theta} = \frac{1}{Hu^2} = \frac{1}{n} \left\{ 1 - 2\delta v - \delta\eta + \frac{9}{2}m^4e'^2 + 3m^2e'\cos n't - \frac{11}{4}m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't) \right. \\ \left. - \frac{77}{8}m^2e'\cos(2\theta - 3n't) + \frac{11}{8}m^2e'\cos(2\theta - n't) \right\},\end{aligned}$$

or

$$\begin{aligned}\frac{ndt}{d\theta} &= 1 + \frac{9}{2}m^4e'^2 + 3m^2e'\cos n't - \frac{11}{4}m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't) \\ &\quad - \frac{77}{8}m^2e'\cos(2\theta - 3n't) + \frac{11}{8}m^2e'\cos(2\theta - n't) \\ &\quad - \frac{425}{24}m^2e'\frac{de'}{ndt}\sin(2\theta - 2n't) + \frac{595}{48}m^2\frac{de'}{ndt}\sin(2\theta - 3n't) \\ &\quad - \frac{85}{48}m^2\frac{de'}{ndt}\sin(2\theta - n't).\end{aligned}$$

Divide by

$$1 + \frac{9}{2}m^4e'^2 + 3m^2e'\cos n't$$

and take into account $m^4e'^2$ in the non-periodic term,

$$\begin{aligned}\therefore \frac{ndt}{d\theta} \left\{ 1 - 3m^2e'\cos n't \right\} &= 1 - \frac{11}{4}m^2\left(1 - \frac{5}{2}e'^2\right)\cos(2\theta - 2n't) \\ &\quad - \frac{77}{8}m^2e'\cos(2\theta - 3n't) + \frac{11}{8}m^2e'\cos(2\theta - n't) \\ &\quad - \frac{425}{24}m^2e'\frac{de'}{ndt}\sin(2\theta - 2n't) \\ &\quad + \frac{595}{48}m^2\frac{de'}{ndt}\sin(2\theta - 3n't) - \frac{85}{48}m^2\frac{de'}{ndt}\sin(2\theta - n't),\end{aligned}$$

and therefore

$$\begin{aligned} \int n dt = \theta + 3me' \sin n't - \frac{11}{8} m^2 \left(1 - \frac{5}{2} e'^2\right) \sin (2\theta - 2n't) \\ - \frac{77}{16} m^2 e' \sin (2\theta - 3n't) + \frac{11}{16} m^2 e' \sin (2\theta - n't) \\ + 3 \frac{de'}{ndt} \cos n't + \frac{295}{24} m^2 e' \frac{de'}{ndt} \cos (2\theta - 2n't) - \frac{413}{48} m^2 \frac{de'}{ndt} \cos (2\theta - 3n't) \\ + \frac{59}{48} m^2 \frac{de'}{ndt} \cos (2\theta - n't). \end{aligned}$$

Hence θ differs from $\int n dt$ by periodic terms only, which proves the proposition.

The value of $\frac{dn}{ndt}$ above found agrees with that found in my paper published in the 'Philosophical Transactions' for 1853.

Note on the Constant of Lunar Parallax. By Professor J. C. Adams, M.A., F.R.S.

From the report of a discussion which took place at a late meeting of the Society, I have reason to believe that an explanation of the apparent discrepancy between the value of the constant of parallax given by me in the Appendix to the *Nautical Almanac* for 1856, and in the *Monthly Notices*, vol. xiii. p. 263, and the value of the constant found by Hansen in the Introduction to his Lunar Tables, may not be unacceptable to some of our members.

It will be proper to begin this explanation by recalling to mind that my formula, in the article of the *Monthly Notices* above referred to, does not represent the parallax itself, but rather the sine of that quantity converted into seconds of arc by dividing by $\sin 1''$ or, which is the same thing, by multiplying by the number of seconds in the arc equal to the radius. The employment of the sine of the parallax instead of the parallax itself appears to be desirable both on theoretical as well as practical grounds.

In the first place, the sine of the parallax, being proportional to the reciprocal of the radius vector, is the quantity given directly by the lunar theory, and, in the next place, it is the same quantity which is wanted in the reduction of lunar observations.

What I have called the constant of parallax in the papers above referred to is, then, the constant term in the expression for the converted sine of the parallax, supposing the periodic